

Problem 2.3

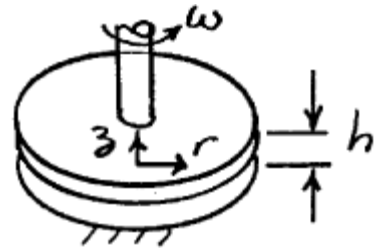
[Difficulty: 2]

2.3 A viscous liquid is sheared between two parallel disks; the upper disk rotates and the lower one is fixed. The velocity field between the disks is given by $\vec{V} = \hat{e}_\theta r\omega z/h$. (The origin of coordinates is located at the center of the lower disk; the upper disk is located at $z = h$.) What are the dimensions of this velocity field? Does this velocity field satisfy appropriate physical boundary conditions? What are they?

Given: Viscous liquid sheared between parallel disks.

Upper disk rotates, lower fixed.

Velocity field is: $\vec{V} = \hat{e}_\theta \frac{r\omega z}{h}$



Find:

- Dimensions of velocity field.
- Satisfy physical boundary conditions.

Solution: To find dimensions, compare to $\vec{V} = \vec{V}(x, y, z)$ form.

The given field is $\vec{V} = \vec{V}(r, z)$. Two space coordinates are included, so the field is 2-D.

Flow must satisfy the no-slip condition:

- At lower disk, $\vec{V} = 0$ since stationary.

$$z = 0, \text{ so } \vec{V} = \hat{e}_\theta \frac{r\omega 0}{h} = 0, \text{ so satisfied.}$$

- At upper disk, $\vec{V} = \hat{e}_\theta r\omega$ since it rotates as a solid body.

$$z = h, \text{ so } \vec{V} = \hat{e}_\theta \frac{r\omega h}{h} = \hat{e}_\theta r\omega, \text{ so satisfied.}$$